Exercise 9

Compute AB, $\det A$, $\det B$, $\det(AB)$, and $\det(A+B)$ for

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & 2 \\ 3 & 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -2 & 0 & 2 \\ -1 & 1 & -1 \\ 1 & 4 & 3 \end{bmatrix}$$

Solution

Compute the product of A and B and their respective determinants.

$$\begin{aligned} AB &= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 0 & 2 \\ -1 & 1 & -1 \\ 1 & 4 & 3 \end{bmatrix} \\ &= \begin{bmatrix} (1)(-2)+(-1)(-1)+(0)(1) & (1)(0)+(-1)(1)+(0)(4) & (1)(2)+(-1)(-1)+(0)(3) \\ (0)(-2)+(3)(-1)+(2)(1) & (0)(0)+(3)(1)+(2)(4) & (0)(2)+(3)(-1)+(2)(3) \\ (3)(-2)+(1)(-1)+(1)(1) & (3)(0)+(1)(1)+(1)(4) & (3)(2)+(1)(-1)+(1)(3) \end{bmatrix} \\ &= \begin{bmatrix} -1 & -1 & 3 \\ -1 & 11 & 3 \\ -6 & 5 & 8 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \det A &= \begin{vmatrix} 1 & -1 & 0 \\ 0 & 3 & 2 \\ 3 & 1 & 1 \end{vmatrix} \\ &= 0 \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} + 3 \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} - 2 \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} \\ &= 0[(-1)(1)-(0)(1)] + 3[(1)(1)-(0)(3)] - 2[(1)(1)-(-1)(3)] \\ &= 0(-1) + 3(1) - 2(4) \\ &= -5 \end{aligned}$$

$$\begin{aligned} \det B &= \begin{vmatrix} -2 & 0 & 2 \\ -1 & 1 & -1 \\ 1 & 4 & 3 \end{vmatrix} \\ &= -2 \begin{vmatrix} 1 & -1 \\ 4 & 3 \end{vmatrix} - 0 \begin{vmatrix} -1 & -1 \\ 1 & 3 \end{vmatrix} + 2 \begin{vmatrix} -1 & 1 \\ 1 & 4 \end{vmatrix} \\ &= -2[(1)(3)-(-1)(4)] - 0[(-1)(3)-(-1)(1)] + 2[(-1)(4)-(1)(1)] \\ &= -2(7) - 0(-2) + 2(-5) \\ &= -24 \end{aligned}$$

Compute the determinant of AB and the determinant of $A+B$.

$$\begin{aligned}
\det(AB) &= \begin{vmatrix} -1 & -1 & 3 \\ -1 & 11 & 3 \\ -6 & 5 & 8 \end{vmatrix} \\
&= -1 \begin{vmatrix} 11 & 3 \\ 5 & 8 \end{vmatrix} - (-1) \begin{vmatrix} -1 & 3 \\ -6 & 8 \end{vmatrix} + 3 \begin{vmatrix} -1 & 11 \\ -6 & 5 \end{vmatrix} \\
&= -1[(11)(8) - (3)(5)] + 1[(-1)(8) - (3)(-6)] + 3[(-1)(5) - (11)(-6)] \\
&= -1(73) + 1(10) + 3(61) \\
&= 120
\end{aligned}$$

$$\begin{aligned}
A + B &= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & 2 \\ 3 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 0 & 2 \\ -1 & 1 & -1 \\ 1 & 4 & 3 \end{bmatrix} \\
&= \begin{bmatrix} -1 & -1 & 2 \\ -1 & 4 & 1 \\ 4 & 5 & 4 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\det(A+B) &= \begin{vmatrix} -1 & -1 & 2 \\ -1 & 4 & 1 \\ 4 & 5 & 4 \end{vmatrix} \\
&= -1 \begin{vmatrix} 4 & 1 \\ 5 & 4 \end{vmatrix} - (-1) \begin{vmatrix} -1 & 1 \\ 4 & 4 \end{vmatrix} + 2 \begin{vmatrix} -1 & 4 \\ 4 & 5 \end{vmatrix} \\
&= -1[(4)(4) - (1)(5)] + 1[(-1)(4) - (1)(4)] + 2[(-1)(5) - (4)(4)] \\
&= -1(11) + 1(-8) + 2(-21) \\
&= -61
\end{aligned}$$